Structural Dynamics from a Clakesch Approach. The impact of the Residential Boom in Spain (1996-2010)

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Abstract.

The paper develops an applied general equilibrium (AGE) model, that is rooted in Classical, Keynesian and Schumpeterian traditions. Thus the acronym "CLA-KE-SCH" or the more usual term "PostKeynesian economics".
The "social accounting matrix" implicit in our AGE model, reminds the Classical universe with different social groups each one characterized by a peculiar expenditure pattern. The burden of debt and other financial variables have an impact on disposable income and final consumption. This proves the convenience of building a "stock-flow consistent model".
Any AGE model has embedded a system of prices and a system of quantities that interact with each other. In our case, the price system is based on the Classical theory of value and distribution, updated by Sraffa in 1960. Commodities leave the factory with a label indicating the cost of production (which includes a "normal" rate of profit on the capital invested). Changes in the actual profit rate and the degree of capacity utilization may affect sectoral investment.
Keynes we take the principle of effective demand which explains the equilibrium level of final output (and its composition) as a multiple of autonomous demand. In the vector of final demand we include the leading sectors playing the role of "locomotives" of the economy.
In the last part of the paper we explain the dynamics of the Spanish economy from 1996 till 2010, using the Symmetric Input-Output table for year 2005. The growth process, driven from residential investment of households funded with cheap mortgage credits, very soon proved to be economically unsustainable.

Keywords: (1) Dynamic input-output models; (2) PostKeynesian AGE models; (3) Stock-flow consistent models; (4) Multiplier-accelerator models; (5) Structural change.

1. Introduction.

This paper lays the foundations for a dynamic and computable "applied general equilibrium model" (AGE) that is useful for policy evaluation and growth analysis. It is rooted in Classical, Keynesian and Schumpeterian traditions ("Cla-ke-sch", for short). The link of these streams of economic though in a logical and useful model is the main goal of the paper.

Classical Political Economy was concerned with the analysis of the processes of production, distribution, consumption and accumulation, that bring about economic growth. The social accounting matrix of *section 2* is a fair reflection of this scheme, where distribution (and redistribution) of income plays a crucial role. We add financial flows and stocks, that, according to postKeynesian, does influence aggregate demand.

Our AGE model aims to explain one of the possible systems of prices and quantities embedded in social accounting matrices (SAM) and input-output tables (IOT), describing an economy during a given period. Reading the first block vertically we get Sraffian prices of production that can also be expressed as Post-Keynesian administered prices (*section 3)*. Reading horizontally we obtain the level and composition of output as a multiple of autonomous demand (*section 4*).

Essentially this is Keynes's principle of effective demand rewritten as a fully disaggregated multiplier-accelerator mechanism. The bulk of household consumption is a constant portion of disposable income, so it can be endogeneized and introduced into the multiplier mechanism. A significant part of productive investment undertaken by firms (the so called "expansionary investment") can also be endogeneized and introduced into the "super-multiplier". Alternatively we can use the accelerator as an "ad hoc" procedure to determine expansionary investment and introduce it into the "multiplicand".

Proper autonomous demand consists in modernization investment by firms, residential investment by households, real public expenditures and exports (*section 5*). The driving force of the most dynamic economies lies in modernization investment, i.e. the introduction of new products and new methods of production by "schumpeterian" innovative entrepreneurs. Residential investment has been the main engine of growth in advanced countries during the last boom (1996-2007). Emergent economies have based their growth on exports to the developed world.

2. A social accounting matrix for a growing economy.

A SAM reflects the transactions between activities, factors and institutions. The advantage of a SAM over national accounts and IOT, is that it takes into account a variety of institutions, and presents, in full detail, not only production activities but also distribution, redistribution and final expenditure. The basic design of a SAM proposed by the United Nations Statistics Division (1993) fits our main purpose perfectly, i.e. the analysis of a growing economy. *Table 1* illustrates the SAM from which our theoretical and empirical analysis will be derived. *Table 2* shows the stocks of productive factors and financial assets that set the basis for the economy and may influence their current behaviour.

Table 1

Table 2

Our starting point is a symmetrical input-output table, where *n* homogeneous ***industries*** represented in each column are producing *n* goods (or basket of goods) represented in the rows.[1] Joint production is possible but has already been removed from the table. All the flows are evaluated at basic prices.

The value added in the production process is divided into wages and profits (operating surplus) and assigned to the ***factors of production***: *labour* and *capital*. In the *satellite accounts* that may accompany the SAM (see *table 2*), it would be fitting to gather information about different types of labour and different capital goods. The fact that such information is not generally available is no excuse for ignoring it, but highlights the necessity of making it readily available.

Among the ***institutions*** we include households (H), Government (G), non-financial corporations or “enterprises” (E), financial institutions (F) and the rest of the world (RW). Each of them can be disaggregated according to the particular interests of the researcher. For our purposes it is useful to separate *households* according to the main source of income or to the level of income. H1 would stand for non-qualified labour; H2 for qualified labour; H3 for managers; H4 for pensioners, and so on. *Enterprises* could be separated by size (small, medium and big); by nationality (local, national or multinational) or by industries (*1* to *n*). The last classification facilitates the analysis of accumulation, but may be too detailed. [2]

The flows among industries, factors of production and institutions are classified in four accounts.

a. ***Inputs and outputs: the production account***. The traditional input-output table is included in the first block of rows and columns of *table 1*. The rows register the proceeds from the sale of outputs; the columns, the expenditure derived from the purchase of inputs. ‘Value added’ (**VA**), i.e. the payment of primary incomes to the factors of production, sets the balance.
b. ***Income and consumption: the current account***. This is divided into two subsets (blocks 2 and 3 of *table 1*). Rows in the second block show the *primary distribution of* income into wages and profits (operating surplus). A vertical reading shows the allocation of VA among institutions: (1) The bulk of wages goes to households; (2) A portion of wages and profits go to government as “social security contributions”; (3) A portion of wages and profits (depending on the outstanding debt of households and enterprises) goes to the financial sector; (4) A portion of the operating surplus is retained in corporations as “reserves” (S_e); (5) The rest profits is distributed to households as “property incomes” (R). Taxes on income, current transfers and other redistributive flows are deal in the cross of the third block A horizontal reading of the third block, after redistribution has taken place, gives information about the disposable income of institutions ($\mathbf{Y_d}$). Reading the third

[1] In the UN National Accounts Manual (1993) it corresponds to a square “commodity by industry table”. Each good is produced exclusively by a specific industry, although such industries may produce other related goods. Industry *j*, for instance, would produce “textiles and shoes” in a given proportion that should be kept constant throughout this analysis.

[2] Some institutions may develop a specific activity. Governments provides social services; financial institutions provide financial services, and so on. As providers of services the activity of these institutions is attached to a concrete industry.

block vertically we observe the use of disposable income between final consumption (**C**) and savings (**S**).

c. ***Accumulation and the capital account***. Savings plus capital transfers (**Tk**) allow institutions to finance their investments in the variety of capital goods gathered in the first horizontal block of the SAM (**I)**. Investment feeds the capital stocks (**K**) that we can trace in *table 2*.

d. ***Flows of finance: the financial account***. The last block of our SAM (that goes beyond the traditional ones) informs about the flows of funds (**FF**) from creditors to debtors. The financial account among agents may be displayed to see the type of assets they use: deposits and other liquid assets called "money"; bank loans; obligations, equity. For our purposes it would be useful to have a sheet of outstanding financial assets at the beginning of the period (*table 2*). The stock-flow consistent models have opened way in this direction. (Godley, 2007).

3. The price system in a capitalist economy.

Competition, in the long run, forces firms to introduce the best available techniques, to use capacity at the optimal level and to adjust prices to production costs (which include a normal and general profit rate on the value of capital invested). Sraffa (1960), following the Classical-Marxian tradition, built a system of equations leading to such prices. He proved that, given the technology available and one of the distributive variables (let's say the "real wage"), we obtain a unique vector of relative prices (in terms of a chosen numeraire) and the other distributive variable (the general rate of profit). For an in-depth analysis of the classical theory of prices see Schefold (1997) and Kurz & Salvadori (1998)

Before expounding the price equations let us comment on what is already 'given', i.e. on technology and distribution.

A) Technology.

Our production functions are taken directly from the first vertical block of the SAM that corresponds to the columns of a symmetrical input-output table (IOT), as we have already explained. They are Leontief's production functions with fixed technical coefficients. Entrepreneurs are free to choose among different techniques, but, once the choice has been made, they cannot combine inputs and factors of production at will. Constant returns to scale are also assumed. In the short run, however, entrepreneurs may change the degree of capacity utilization (the "capital / output" ratio) in an attempt to adjust to demand fluctuations. But using capital more hours a day implies hiring extra labour, so the degree of mechanization (the "capital / labour" ratio) remains constant.

Technology is materialized in the following sets of data.

(a) A matrix of technical coefficients: $A = \Lambda \cdot \hat{q}^{-1}$, Λ being a square matrix for intermediate consumptions or inter-industry transactions and q the (column) vector of the total value produced in each industry. The result is a square matrix $n \cdot n$, n being the number of homogeneous industries. We should separate domestic from imported intermediate tables (Λ_d, Λ_m) and compute two different matrices of technical coefficients (A_d, A_m). Let $\hat{\theta}$ be a diagonal matrix indicating the percentage of each good that is imported. These percentages may change either when there is a variation in the exchange rate or in the ratio demestic price to international price. Our previous matrix A should be segmented into two: $A_m = A \cdot \hat{\theta}$ and $A_d = A \cdot \left\langle Id - \hat{\theta} \right\rangle$. [3]

(b) A rectangular matrix of labour coefficients: $l = L \cdot \hat{q}^{-1}$. L is part of the satellite accounts. It has as many columns as industries and as many rows as types of labour: non-qualified labour, qualified labour, managers … *l* will have the same dimensions. It is an inverse measure of direct labour productivity.

(c) A rectangular matrix of capital coefficients: $k = K \cdot \hat{q}^{-1}$. *K* is the capital matrix with a column for each industry and a row for each capital good (buildings, equipment, industrial vehicles and so on). We could represent *K* as a square matrix, but only the rows corresponding to capital goods would have positive figures. *k* has the same dimensions and content as *K* but refers to a unit of production. The figures in *k* refer to the *normal* or *desired* "capital / output" coefficients.

B) Distribution.

Workers consider the real wage achieved in the past (w_r) as a social conquest. In yearly wage agreements, trade unions try to consolidate it, fixing the nominal wage (w) so that: $w_{r,t}=w_{t-1}+\tau$, (τ being the expected inflation for year t, which can be proxied by the inflation rate targeted by the central bank). Historically, the real wage has risen *pari passu* with productivity, so we can expect trade unions to claim for increases in the nominal wage in proportion to productivity gains, although it make take several periods to achieve this. Tensions in the labour market (reflected by percentage deviations of the employment rate *(ε)* from its conventional level) will encourage trade unions to demand further increases in nominal wages. [4]

[3.1] $$w_t = w_{t-1} + \tau + f(\hat{\pi}) + f'(\hat{\varepsilon})$$

In the previous equation w_t refers to the nominal wage in year *t* for the basic labour category (let's say, 'non-qualified labour'). Other types of labour will earn a multiple of this.

In Classical political economy, profits appear as an "operating surplus" belonging to the owners of capital. The role of prices of production is to distribute such surplus among industries in such a way that the representative firm of each industry gets the same rate of profit on the capital invested (*r*). [5] Note that at this point we depart from the major Sraffian tradition that takes the rate of profit (instead of the real wage) as

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[3] In this paper *Id* stands for the identity matrix. A diagonal matrix is represented either by a circumphlex (^) or angular brackets (< >)

[4] Alternatively, we could refer (after changing the sign) to deviations from the conventional unemployment rate. Note, however, that there is no 'natural' employment rate (or unemployment rate) determining a long period equilibrium. It is just a historical position that is bound to change with aggregate demand fluctuations.

[5] We assume that only fixed capital is properly advanced. Intermediate consumption and wages are paid regularly out of sales proceeds. By 'representative' firms we mean the ones using the best available technology. Probably a handful of innovative firms are using more productive technologies protected by patents and the like. Other firms may be using old-fashioned technology until they replace capital or quit the industry. Note, that whenever we compute technological coefficients by calibration from an IOT, we obtain the average technology in the industry.

a datum, identified with the interest rate (Pivetti, 1991). In our model both the interest rate and the exchange rate are taken as given (acknowledging the influence of Central Bank's monetary policy). But the interest rate does not regulate the rate of profit. A fall in interest rates will lead to a fall in property incomes, a fall in dividend payments (which are usually linked to the interest rate) and a rise in non-distributed profits (corporations' savings).

To obtain the disposable income of households and other institutions we still need to add or subtract current transfers and direct taxes. The "effective" tax rates on different incomes can be obtained by calibration of the data gathered in table *T* of our SAM.

C) Prices of production and "administered" prices.

Now we are ready to derive the Sraffian system of prices of production corresponding to a competitive capitalist economy. It results from a vertical reading of the matrices of technical coefficients and value added coefficients. The price of production of any commodity can be represented as a multiple of the unit costs of "non produced inputs", i.e. labour ($w\cdot l$) and imported inputs ($p_m.A_m$)[6]. The price-multiplier appears as an inverse matrix that diffuses any shock throughout the industrial structure.

[3.2]
$$\begin{aligned} p &= p\cdot A + w_r\cdot l + r\cdot(p\cdot k) = \\ p &= p\cdot A_d + p_m\cdot A_m + w_r\cdot l + r\cdot(p\cdot k) = \\ p &= (w_r\cdot l + p_m\cdot A_m)\cdot[Id - A_d - r\cdot k]^{-1} \end{aligned}$$

The problem with the preceding account is that we have to rely on a fixed capital matrix (*k*) that generally does not exist or is not fully reliable. For these reason, some economists refer profits to circulating capital (intermediate consumption and wages). Our previous term $r\cdot(p\cdot k)$ would be replaced by $b\cdot P$, where *b* stands for a diagonal matrix of sectoral *profit shares* and *P* is the vector of "administered" or "mark-up" prices. The result will be the same, provided the profit share is proportional to the sectoral intensity of capital. Industries with higher capital / output ratio are supposed to have a higher profit share.

[3.3]
$$\begin{aligned} P &= P\cdot A_d + p_m\cdot A_m + w\cdot l + \hat{b}\cdot P \\ P &= (w\cdot l + p_m\cdot A_m)\cdot\left[Id - A_d - \hat{b}\right]^{-1} \end{aligned}$$

By construction, the vector of prices we obtain will be the unit one: [1, 1, …1]. This is because we have obtained technical coefficients and value added shares dividing each column of the IOT by total output (in value terms). Despite such an odd result, there should be no problem in computing the impact on prices of a change in wages, tariffs, productivity and so on. We should be careful, however, with the way we represent the shock and the transmission mechanism. A 10% increase in wages (*w*) will bring about a 10% in all prices, if the rate of profit (*r*), the profit margin (β) and the

[6] Prices of imports (p_m) include tariffs and taxes related to foreign trade. Value added tax (VAT) could be included as an additional row. Such a representation would facilitate the analysis of the impact on market prices of a change in VAT. Since we have decided to use basic prices, VAT is included in table *T* and is computed as a fraction of final consumption.

profit share (b) keep constant. This describes an inflationary process where nominal wages are increasing but the real wage remains constant. Whenever we want to simulate a rise in the real wage (w_r) we ought to compute the movements in relative prices, fixing p_1=1 and allowing r, β and b to fall. Technical progress would cause a positive effect on r unless real wages rise instantaneously absorbing all productivity gains.

D) Market prices and demand fluctuations.

So far we have just considered supply forces. Markets prices are supposed to reflect both supply and demand tensions. In principle, excess in demand will push prices up. This is a transient phenomenon since higher prices and profits in industry j will attract investment and production will rise cancelling out the excess of demand in the output of j. After the adjustment of quantities, relative prices will return to the long run equilibrium determined by production costs.

This is the theory behind all of this. In practice only a handful of primary products (oil and raw materials, in particular) are sensitive to demand shocks, as Post-Keynesians have repeatedly shown after Kalecki (1971) and Sylos-Labini (1957). Such prices are determined abroad and are taken as data in our model. In an advanced industrial economy, the bulk of industries is prepared to accommodate demand shocks by piling inventories and adjusting capacity utilization. In services there is no such possibility, but the risk of losing customers by continuous changes in prices has convinced entrepreneurs to maintain prices in their long run equilibrium, determined by costs of production.[7] We can take it for granted –a key hypothesis for our purposes– that in an advanced industrial economy relative prices are rarely influenced by the ordinary ups and downs of demand.

4. Induced demand.

To make the expected increases in autonomous demand feasible, firms are supposed to demand:
(a) Intermediate goods or “circulating capital” (this is the *intermediate consumption* that is given by IOT);
(b) Labourers who will consume a significant proportion of their new incomes in a variety of consumption goods (*induced final consumption*, computed in section B, below).
(c) Capital goods to substitute or repair the machinery used up during the process of production (*fixed capital consumption*, computed in section A);
(d) Capital goods to enlarge capacity in order to attend efficiently the expected increases in demand (*expansionary investment*, computed by the acceleration mechanism in section C)

A) Induced consumption of fixed capital

The FCC so computed is allocated among the three capital goods and among sectors according to the weight of each capital good (K_i) in the total stock of capital (K). We

[7] Neo-Keynesian literature explains this phenomenon under the heading of ‘menu costs’.

also consider the speed of depreciation of each capital good, (an inverse measure of the number of years that each capital good is considered to endure, n_i)[8]. From the following formula we compute the parameter *a'* which ensures that the whole value of FCC is allocated into our 12 sectors.

[4.1]
$$FCC(ag) = \left(\frac{K_1}{K} \cdot \frac{1}{n_1} + \frac{K_2}{K} \cdot \frac{1}{n_2} + \frac{K_3}{K} \cdot \frac{1}{n_3} \right) \cdot a'$$
$$a' = \frac{FCC(ag)}{(.)}$$

Each cell of the FCC matrix is computed multiplying *a'* by the capital share corresponding to each sector and good.

[4.2]
$$FCC = \begin{bmatrix} \frac{K_{31}}{K_3} \cdot \frac{1}{n_3} \cdot a' & \frac{K_{32}}{K_3} \cdot \frac{1}{n_3} \cdot a'(...) & 0 \\ \frac{K_{41}}{K_4} \cdot \frac{1}{n_4} \cdot a' & \frac{K_{42}}{K_4} \cdot \frac{1}{n_4} \cdot a'(...) & 0 \\ \frac{K_{51}}{K_5} \cdot \frac{1}{n_5} \cdot a' & \frac{K_{52}}{K_5} \cdot \frac{1}{n_5} \cdot a'(...) & \frac{K_{5,12}}{K_5} \cdot \frac{1}{n_5} \cdot a' \end{bmatrix}$$

FCC is a $12 \cdot 12$ matrix, although only the rows corresponding to industries producing capital goods (3, 4 and 5) are filled. All the cells of the households sector (12th column) are nil except the one corresponding to the construction industry (5th row). There we include the depreciation of houses owned by families.

Adding up ***FCC*** to the ordinary "transaction table" we obtain the extended matrix of "circulating capital" (***CC***). It includes intermediate consumption, fixed capital consumption and induced final consumption. A multiplier model is mostly interested in the circulating capital produced in the country ($\boldsymbol{CC_d}$). It is obtained by subtracting imports from total transactions. (The information is provided by the original TIO which distinguish among total, domestic and imported quantities).

B) Induced final consumption.

In *The General Theory*, Keynes assumed that the bulk of private consumption depended on household disposable income. This hypothesis was verified at that time and has been ratified ever since. Kalecki contributed to the debate suggesting that the aggregate propensity to consume was a weighted average of the propensities of different income groups. Our SAM allows us to represent different social groups, each one characterized by a particular propensity to consume and a particular consumption basket. Following Dejuán, Cadarso & Córcoles (1994), we can derive induced consumption in two o three steps.

The first step consists in computing disposable income of households. We can derive it directly from the SAM. By calibration we could obtain tax rates, percentage of distributed profits that will allow the passages from value added by factors (*VA*) to

[8] According to IVIE (2009) the amortization period reaches 14 years for vehicles (industry 3); 11,25 years for machinery and equipment (industry 4); 44,28 years for industrial constructions; and 60 years for dwellings owned by households.

disposable income of institutions (Y_{d1}). At this moment we are only interested in the first set of columns of Y_d which informs of the disposable income of the households.

Pre-multiplying by <PC> we obtain the incomes that are systematically consumed. In the diagonal of <PC> we find the consumption propensities of the different income groups (*h*). [DC] indicates the distribution of consumption expenditure among goods.[9] By construction, any column of [DC] adds up to 1. To obtain *domestic* induced consumption [$C_{i,d}$] we have to subtract the portion of consumption goods imported from abroad. From this figure we remove value added tax (<*Id-VAT*>) in order to get *effective* consumption.

[4.3]
$$\begin{aligned}[C_i]_{nn} &= [CD]_{nh} \cdot \langle CP \rangle_{hh} \cdot [Y_d]_{hn} \\ [C_{i,d}]_{nn} &= \langle Id - VAT \rangle_{nn} \cdot \langle Id - \hat{\theta} \rangle_{nn} \cdot [C_i]_{nn}\end{aligned}$$

Our model is ready to introduce the influence of prices in the distribution of consumption among different goods or the influence of interest rates on the consumption (and saving) propensities. We are not going to do so because, empirically, these new variables add very little to the consumption function. The Cambridge multisectoral model has shown that linear expenditure functions, similar to the ones we have used here, explain consumption better than any other. (Barker & Petterson, 1987, following Stone's suggestions; Stone, 1981). Changes in prices might affect the substitution in consumption between, let's say, two different types of meat, but not between food and clothing, which is the level we are considering in a SAM.

C) Expansionary investment (the "accelerator").

According to the acceleration principle, *expansionary investment* depends on the expected growth of income. Its exact value may be computed multiplying the expected rate of growth of the economy for the capital installed. (***K***).

[5.1]
$$I_{(12\cdot 1),(t)} = K_{(12\cdot 12),(t)} \cdot g^{e}_{(1\cdot 12),(t)}$$

$\boldsymbol{I_{(t)}}$ is the investment column vector at the end of period *t*. It refers to the capital goods that will be produced during year t and, presumably, will be purchased by firms at the end of the year (31/12/t). $\boldsymbol{K_{(t)}}$ informs about the stocks of each capital good in each sector at the beginning of period *t* (01/01/t). $\boldsymbol{g^e}$ stands for the expected rate of growth of each commodity. In the usual uncertainty that characterizes private decisions, the expected growth for the current year (*t*) is proxied by the actual rate of growth of the economy in the previous period ($\boldsymbol{g_{(t-1)}}$). Errors will show up in excesses of capacity (positive or negative) ($\boldsymbol{EK_{(t-1)}}$). Later on, they will be subtracted or added to the investment derived form the acceleration principle ($K_{(12\cdot 12)(t)} \cdot g_{(12\cdot 1)(t-1)}$) in order to approach the desired "capital/output" ratio in each sector.

[9] Information about propensities to consume and expenditure patterns can be obtained from family budget statistics. The problem is that the consumption functions of such statistics do not coincide with the consumption goods contemplated in input – output tables. Something else is needed to link both sets of statistics. Econometrics will help to fill up certain gaps.

[5.2]
$$
\begin{aligned}
I_{(12\cdot 1),(t)} &= K_{(12\cdot 12),(t)} \cdot g_{(12\cdot 1),(t-1)} - EK_{(12\cdot 1),(t-1)} \\
K_{(12\cdot 12),(t)} &= K_{(12\cdot 12),(t-1)} + I_{(12\cdot 12),(t-1)} \\
EK_{(12\cdot 1),(t-1)} &= KR_{(12.12),(t-1)} \cdot i_{(12\cdot 1)} - K_{(12\cdot 12),(t-1)} \cdot i_{(12\cdot 1)} \\
KR_{(12\cdot 12),(t-1)} &= \left(k_{(12\cdot 12),} \cdot q_{(t-1)}\right) \cdot \left\langle g_{(t-1)} \right\rangle
\end{aligned}
$$

$\boldsymbol{EK}_{(t-1)}$ gathers the excesses of capacity. It results from subtracting installed capital in period *(t-1)(**K**)*, from required capital in the same period (***KR***). (Both stocks appear in matrix form; ***i*** is a column vector of ones which adds up the value of the rows of capital matrices). Required capital ($\boldsymbol{KR}_{(t-1)}$) results from multiplying the desired capital/output ratios (***k***) times net output in (t-1), times effective rates of growth of sectoral output in (t-1). Installed capital in any year *t* results from adding net investment at the end of period *t* to the stock of capital installed at the beginning of the same period (see the second equation of [5.2], where net investment is presented in matrix form).

The path of investment may be also altered by changes in the rate of profit. In prices and profits rates go up in a given industry, saving will accrue massively towards it. Presumably the excess of demand will disappear, prices will return to their normal position (cost of production) and the rate of profit will be in line with the general one.

5. The quantity system in motion.

In *section 3* we saw that a vertical reading of the first block of the SAM may lead to the Sraffian system of prices of production. We can expect that competition in a capitalist economy will enforce such prices. In a similar vein, a horizontal reading of the SAM would provide a system of quantities. This was implicit in the Classical equations (Shefold, 1997; Kurz and Salvadori, 1998; Nell, 1998, 2004). And it was the core of the von Neumann (1945-46) general equilibrium model. Both approaches provide useful hints for understanding certain equilibrium conditions and certain technological limits. But they do not adequately describe the working of a capitalist economy that is supposed to be a *demand-constrained system*. According to the Keynesian *principle of effective demand*, the equilibrium level of output at any given moment does not depend on the productive capacities of the economy but on the expected demand at normal prices (Keynes, 1936; Kalecki, 1971; Kornai, 1979). It can be expressed as a multiple of the autonomous demand expected for the period under consideration.

Production will adjust to aggregate demand (autonomous plus induced). Output will rise until the new ‘uncommitted incomes’ ($\boldsymbol{\Sigma}$ = incomes not devoted systematically to induced consumption) match the value of autonomous demand (Z^*). *Table 3* clarifies this process and emphasizes that it is Z^* which determines $\boldsymbol{\Sigma}$, in the same way Keynes proved that it is *I* which determined *S*.

Table 3

A) The multiplier.

The transformations entered in sections (4.a) and (4.b) allow us to compute the multiplier that links all types of induced demand. First we obtain the enlarged inter-industry transaction table (Λ^*), adding up the tables of intermediate consumptions (Λ_d) and final induced consumption (C_{id}). Second, we divide the cells of each column by the total output of the industry to obtain the enlarged matrix of coefficients (A^*_d). Then we compute a Leontief's inverse matrix to obtain the multiplier of total output (MQ).

[5.1] $$\Lambda^*_d = \Lambda_d + C_{i,d}$$

[5.2] $$A^*_d = \Lambda^*_d \cdot \hat{q}^{-1}$$

[5.3] $$MQ = \left[I - A^*_d\right]^{-1}$$

Economists are generally less interested in total output (it involves the problem of double counting of intermediate goods) than in final output, equal to income or value added. They are more interested in employment. *Table 4* explains how to obtain the corresponding super-multipliers of income (*MV*) and labour (*ML*).

[5.4] $$MV = v \cdot \left[I - A^*_d\right]^{-1}$$

[5.5] $$ML = l \cdot \left[I - A^*_d\right]^{-1}$$

Each column of any of the super-multiplier matrix informs us about the direct and indirect effects of a unitary expansion of industry *j* over the output, income or employment of all the industries providing resources to *j*. Provision may be direct or indirect, and the "resources" are defined in the broadest sense so as to include intermediate goods, final consumption goods demanded by new incomes, and fixed capital goods to expand capacity at the required rate.

B) The multiplicand

The AGE model so far sketched allows us to compute the level of output and employment at a given moment and their increases after a supply or demand "stimulus". Output at year *t* can be presented as a multiple of the expected autonomous demand in that year. We shall refer to it as Z_t which includes proper autonomous demand (Z_t) and expansionary investment computed by the acceleration principle (I_t).

An increase in any of the components of the (column) vector of autonomous demand will bring about an increase in output, compounded by the structural multiplier.

[5.6] $$\Delta q_t = \left[MQ\right] \cdot \Delta Z^*_t$$

Similar expressions can be found for income and labour, applying the corresponding multipliers (*MV* and *ML*).

[5.7] $$\Delta y_t = [MV] \cdot \Delta Z_t^*$$

[5.8] $$\Delta L_t = [ML] \cdot \Delta Z_t^*$$

In the analysis of a dynamic economy, the rates of growth of autonomous demand (vector g_z) are the key element, since induced demand adapts passively to the former. What does this vector contains? As a general rule, autonomous demand should include expenditure not funded by national income and expenditure that is not systematically related to national income.

a. *Productive investment of the expansionary type*. The peculiarity of this element is that it can be computed *ad hoc* via the *accelerator* mechanism. We could add it to induced final consumption in order to derive a "supermultiplier"[10]. Its dependence on a variable so volatile as the "expected rate of growth of demand" suggests, however, that we'd better include it in the "multiplicand".
b. *Modernization investment* by innovative entrepreneurs. It is the main vehicle of technical change. Our "sequential" AGE model allows to change year by year technical coefficients.
c. *Residential investment* by households. It has been the basic engine of growth in the last boom (1996-2007).
d. *Public expenditures* in goods and services. It is a major level for stabilizing the economy.
e. *Exports.* It is the key element of small open economies.

6. Conclusions

For a given economy in a given moment we can compute the potential rate of growth. The actual rate of growth will depend on the expected rate of growth of autonomous demand whose main ingredients are: modernization investment, residential investment and exports. They are the actual engines of growth or the "gas pedals". The rate of growth and the type of growth will depend on the pedal than entrepreneurs are pushing in each moment. Each pedal has different "drawing effects". They depend on the induced investment (captured by the acceleration mechanism) and the induced consumption (captured by the income and employment multipliers).

Only input-output models give such a broad and deep vision of the growth process in a capitalist economy. Social accounting matrices inserted in a stock-flow consistent model, are able to capture the feed back between the maladjustments of of physical capital and the burden of financial assets. If we add a proper theory of prices we can analyze the impact of costs on prices and the profit rate, that may influence consumption and investment.

[10] The theoretical basis of the super-multiplier model are explained in Hicks (1950), Serrano (1995), Trezini (1995) and De-Juan (2004).

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Table 1: SAM

		1' **PRODUCTION** **<u>Purchase of inputs</u>**	**2'** **INCOME** **<u>Secondary distribution</u>**	**3'** **CONSUMPTION** **<u>Use of income</u>**	**4'** **ACCUMULATION** **<u>Investment & X'</u>**	**5'** **FINANCE** **<u>Lending</u>**	**total**
		1, 2, .. n	L(W), K(B)	H; G; E; F; W	H; G; E; F; W	H;G;E;F;W	
1. PRODUCTION **<u>Sale of output</u>**	1 2,… n	Intermediate Consumption **Λ**		Final Consumption **C**	Final Investment **I**		tot. outp T1
2. INCOME **<u>Primary Distribution.</u>**	L(W) K(B)	<u>Value Added</u> **VA**					T2
3. CONSUMPTION **<u>Disposable income</u>**	H G E F W		Income institutions (interests, profits h) **Y** <u>Disposable income</u>, **Yd**	Current transfers (includes d. taxes) **T**			T3
4. ACCUMULATION **<u>Savings</u>**	H G E F W			<u>Savings</u> **S**	Capital Transfers **Tk**		T4
5. FINANCE **<u>Borrowing</u>**	H G E F W				<u>Net lending</u>	Flows of Financial Funds **FF**	T5
Total		Total inputs, T1	T2	T3	T4	T5	

Table 2. Satellite accounts of flows and stocks. (And significant economic ratios)				
		Indus-tries	**Institu-tions**	**Aggre-gate**
1	**Employment (L)** - non qualified labour - qualified labour	 x x		 x x
	Rate of employment and/or unemployment *Impact on nominal wages and on the price index*			x
2	**Stock of fixed capital (installed capacity) (K_t=K_{t-1}+I_t)** - construction - equipment - industrial vehicles.	 x x x	 x x x	 x x x
	Excess of capacity ($Ek_t = K_t - K_{r,t}$ = capacity installed – capacity required) **Degree of utilization** ($u_t = K_{r,t}/K_t$) *Expansionary investment tries to correct the disequilibria showed in Ek or u*	x x		x x
	Rate of profit Spread from the average. It may accelerate investment	x		x
3	**Stock of financial assets (FA)** *Stock which is fed with the flows of net lending (see FF in SAM. In the last column foreign debt and assets* Money (deposits and other liquid assets) Debt with bancs Debt (non banc) Equity		 x x x x	 x x x x
4	**Stock of natural Resources** *Quantity and quality of given resources.*	x		x
	Flow of emissions	x		x

Table 3. A compact dynamic SAM.			
		INDUCED DEMAND	AUTONOMOUS DEMAND
		1, 2, ... n, h	H, E, G, W
COMMITTED INCOMES	1 2 … n h	**Λ^*** Intermediate consumption. Fixed capital consumption. Induced final consumption.	**I** Expansionary investment (E) --------------------------------------- **Z** Modernization Inv. (E) Residential investment (H) Autonomous Consumption (H) Government expenditures (G) Exports (W)
UNCOMMITTED INCOMES	H E G W	**Σ** (Z+I)→Σ	